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Model Reduction for Flexible Structures: Test Data Approach

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Introduction

OBTAINING a reasonable reduced model for a flexible structure (a system with separated complex poles and small damping) is an important task for an analyst as well as a test engineer. This task has been satisfactorily solved for analytical models of a flexible structure; however, the reduction of a model of a flexible structure obtained from test data has not yet been considered. In this Note, two important questions are solved that can help one to use resonance test data in model reduction.

A reduced model is obtained by truncating part of the state variables. The reduction indices determine which component is deleted or retained in the reduced model. The indices are obtained from the transfer function (in the form of resonance test data), rather than from the system matrices.

Test data, besides system dynamics, also include actuator and sensor dynamics. The reduced model obtained from these data can be far from the optimal because it includes unwanted actuator-sensor dynamics. In this Note, the reconstruction of the flexible structure indices from the joint actuator-sensor-flexible structure indices is discussed and illustrated.

Reduction Indices from Test Data

There are two indices used in model reduction: Hankel singular values of Moore[1-4] and component costs of Skelton.[3-6] The Hankel singular value γ_i, a simultaneous measure of controllability and observability of the ith state coordinate, is determined in the balanced coordinates. In this case the reduced system is obtained by deleting the least controllable and observable states.

Component cost σ_i is a norm of the ith component. For uncoupled coordinates, the norm of the output is a sum of norms of each component. This approach has proved to be quite successful when modal coordinates, which are almost always uncoupled, are used.

Received April 12, 1989; revision received Oct. 25, 1989.

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It is well known that flexible structures with small damping and separated poles have controllability and observability grammians that are diagonally dominant[1] [$W_c \approx \mathrm{diag}(W_{ci})$, $W_0 \approx \mathrm{diag}(W_{0i})$], where, from Ref. 3,

$$W_{ci} = I_2(||b_i||^2/4\zeta_i\omega_i), \qquad W_{0i} = I_2(Y_i^2\zeta_i\omega_i/||b_i||^2) \tag{1}$$

and Y_i is the norm of the output at ith natural frequency (see the Appendix). For these structures, the balanced grammians are obtained simply as $W_{cb} = W_{0b} = (W_cW_0)^{1/2} = \mathrm{diag}(W_{bi})$, and from Eq. (1) one obtains $W_{bi} = I_2Y_i/2 = I_2Y^2$. This shows that for flexible structures the square of the ith Hankel singular value is equal to one-half of the norm of the output at the ith resonance:

$$\gamma_i = 0.5Y_i \tag{2}$$

Figure 1 shows the determination of γ_2 from the resonance test data for a single-input/single-output system:

$$\gamma_2 = 0.5Y_2 = 0.5||H(\omega_2)||$$

The cost of the ith modal coordinate is determined from Ref. 4:

$$\sigma_i^2 = \mathrm{tr}(A_i\gamma_i^2) = 0.5\zeta_i\omega_iY_i^2 \tag{3}$$

where A_i is given in the Appendix. Denoting the half power frequency[7] $\Delta_i = 2\zeta_i\omega_i$ (see Fig. 1), one can rewrite Eq. (3) as

$$\sigma_i = 0.5Y_i\sqrt{\Delta_i} = \gamma_i\sqrt{\Delta_i} \tag{4}$$

Unlike Hankel singular values, the cost consists of a product of the resonance amplitude and the resonance width.

As an example, the flexible structure in Fig. 2 with 42 state variables, single input and single output, is considered. Sensors and actuators are placed so that all Hankel singular values are equal ($\gamma_i = 1$, $i = 1,\ldots, 42$); i.e., all states are equally controlled and observed. The plots of Hankel singular values are given in Fig. 3. The analytical Hankel singular values (solid line) and those obtained from the transfer function (dotted line) are nearly the same, especially for $nr < 16$.

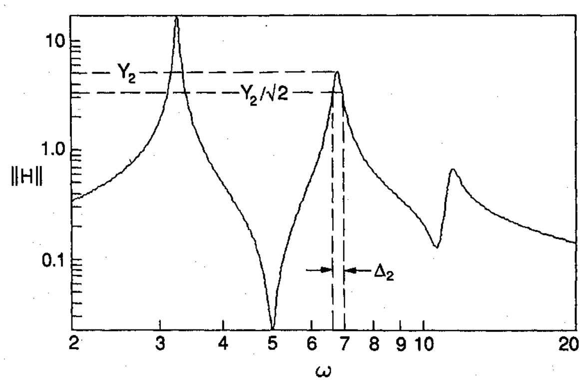

Fig. 1 Hankel singular value and component cost from resonance test.

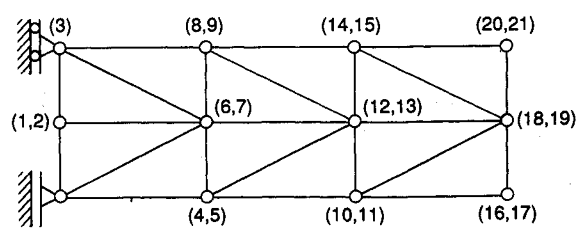

Fig. 2 Flexible structure.

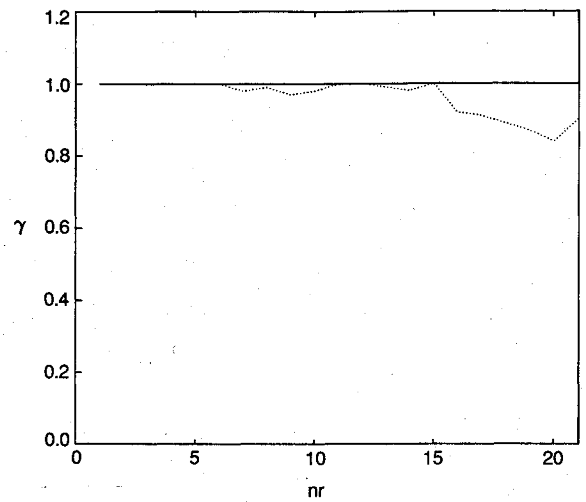

Fig. 3 Hankel singular values: analytical and from the resonance test.

Effect of Actuator Dynamics on the Reduction Indices

The case of combined actuator-sensor-flexible structure dynamics and sensor-flexible structure dynamics can be considered a special case of the actuator-flexible structure dynamics problem; therefore, only the latter is considered in this Note. The system considered is a cascade connection of actuators and a structure. Its transfer function H is as follows: $H = H_sH_a$, where H_s and H_a are transfer functions of the structure and actuators, respectively. Define

$$\cos\phi = ||H||/(||H_s||\ ||H_a||) \tag{5}$$

(the Froebenius norm is used); then, from the preceding definition,

$$||H_s|| = ||H||/(||H_a||\ \cos\phi) \tag{6}$$

The value of $\cos\phi$ determines the collinearity of H_a and H_s. If the actuator transfer function is almost collinear with the structure transfer function, then $\cos\phi \approx 1$. Note that reasonable actuators have $\cos\phi \approx 1$, since orthogonal H_a and H_s result in $H = 0$. Since $||H_s(\omega_i)|| = Y_{si} = 2\gamma_i$ and $||H(\omega_i)|| = Y_i = 2\gamma_i$, then from Eq. (6) it follows that

$$\gamma_{si} = \gamma_i/\alpha_i, \qquad \alpha_i = ||H_{ai}||\cos\phi_i \tag{7}$$

and $H_{ai} = H_a(\omega_i)$, $\cos\phi_i = \cos\phi(\omega_i)$, with ω_i the ith resonance frequency. The Hankel singular values of the structure are obtained from the Hankel singular values of the structure with actuators by scaling the latter by the amplitude of actuators output at the ith resonance frequency and collinearity measure of H_a and H_s.

Next, assuming that $||H_a(\omega)||$ has no abrupt changes in the neighborhood of the structure's natural frequencies, we have $\Delta_i \approx \Delta_{si}$; therefore, the component cost is similarly scaled as the Hankel singular values:

$$\sigma_{si} = \sigma_i/\alpha_i \tag{8}$$

Both Eqs. (7) and (8) show that all Hankel singular values and costs of structure alone are obtained by individual scaling of each related Hankel singular values and cost of the structure-actuator system. If $\cos\phi_i$ is constant for $i = 1,\ldots, n$, then the squares of Hankel singular values and the costs are scaled by the inverse of the actuator gain.

The determination of $\cos\phi$ from Eq. (5) using test data is difficult, if not impossible, since H_s is not known explicitly during testing. However, $\cos\phi_i$ can be determined in a different way, using an expression for H_s from the Appendix. Introducing Eq. (A3) into Eq. (5), one obtains

$$\cos\phi_i = ||c_{si}b_{si}H_{ai}||/(||c_{si}b_{si}||\ ||H_{ai}||) \tag{9}$$

Here, only the actuators transfer function and actuator-sensor locations are involved. For a single-input system, $\cos\phi_i = ||c_{si}H_{ai}||/(||c_{si}||\ ||H_{ai}||)$, and for a single-input/single-output system, $\cos\phi_i = 1$.

As an example, the truss from the preceding section is considered. An actuator with the transfer function $H_a = 10/(s + 2000)$ is applied. Transfer functions of actuator and truss in a series connection are plotted solid line, in Fig. 4, of truss—dotted line and of actuator—dashed line. The Hankel singular values of the truss are all equal to 1 (see Fig. 5, solid line), whereas Hankel singular values of the truss and actuator connection are plotted in this figure by a dotted line. The reconstructed truss Hankel singular values from truss plus actuators Hankel singular values are represented by the dashed line (it overlaps most of the solid line) and indicate the good reconstruction results.

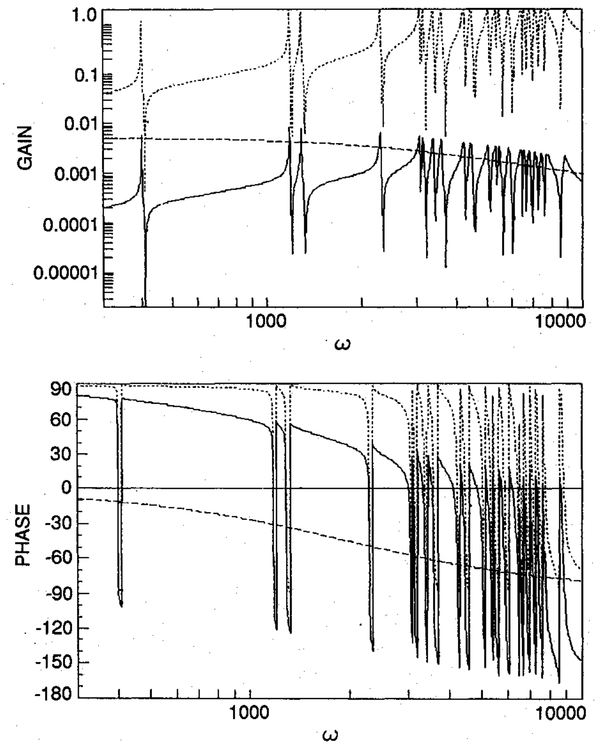

Fig. 4 Transfer functions of the truss (dotted line), the actuator (dashed line), and the truss-actuator cascade connection (solid line).

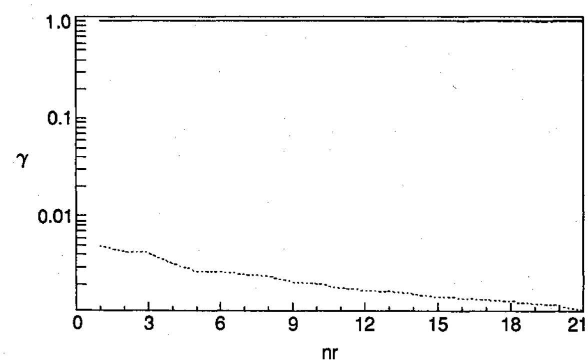

Fig. 5 Hankel singular values of the truss (solid line), the trust-actuator cascade connection (dotted line), and the reconstructed Hankel singular values of the truss (dashed line).

Conclusions

In this Note, methods for the determination of reduction indices from the resonance test data of a flexible structure are presented. If the actuators are in a cascade connection with a flexible structure, the reduction indices of the structure are reconstructed from the indices of the actuator-structure joint dynamics.

Acknowledgment

This work was done while the author held a National Research Council-NASA Langley Research Center Senior Research Associateship.

Appendix

For a flexible structure in modal coordinates $A = \text{diag}(A_i)$, $B^T = [B_1^T, \ldots, B_n^T]$, $C = [C_1, \ldots, C_n]$, and

$$A_i = \begin{bmatrix} -2\zeta_i\omega_i, & \omega_i \\ \omega_i & 0 \end{bmatrix}, \quad B_i^T = [b_i^T \; 0] \quad C_i = [c_{ri} \; c_{di}/\omega] \tag{A1}$$

where ω_i, ζ_i are modal frequency and damping, respectively; $\boldsymbol{b}_i$ is the row vector of actuator locations; and $\boldsymbol{c}_{di}$, $\boldsymbol{c}_{ri}$ are column vectors of displacement and rate sensors locations, respectively. For the transfer function $H = C(j\omega I - A)^{-1}B$ one obtains from Eq. (A1)

$$H = \sum_{i=1}^{n} H_i, \qquad H_i = C_i(j\omega I - A_i)^{-1}B_i \tag{A2}$$

Since for $\omega = \omega_i$ one has $||H_k|| \ll ||H_i||$, for $i \neq k$, therefore, one has $H(\omega_i) \not\approx H_i(\omega_i)$. Next, from Eq. (A1), after some algebra one finds that

$$H_i = a_i \boldsymbol{c}_i \boldsymbol{b}_i \tag{A3}$$

where $a_i = 1/(2\zeta_i\omega_i^2)$, $c_i = \omega_i c_{ri} - jc_{di}$. Define $\cos\Psi = ||c_i b_i|| / (||c_i||\,||b_i||)$, which is a measure of collinearity of c_i and $\boldsymbol{b}_i$; then the norm of H_i, denoted Y_i, is the norm of the output at frequency $\omega = \omega_i$:

$$Y_i = ||H_i(\omega_i)|| = a_i||\boldsymbol{b}_i|| \cos\Psi\sqrt{(||\boldsymbol{c}_{di}||^2 + ||\boldsymbol{c}_{ri}||^2\omega_i)}$$

$$= a_i||\boldsymbol{b}_i||\boldsymbol{c}_i|| \cos\Psi_i \tag{A4}$$

Note that for orthogonal c_i and $\boldsymbol{b}_i$ one obtains $H_i(\omega_i) = 0$; also, $\cos\Psi_i = 1$ for a single-input as well as for a single-output system and for collocated sensors and actuators.

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Angle-Only Tracking Filter in Modified Spherical Coordinates

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I. Introduction

THE Kalman filter by Hoelzer et al.[1,2] for planar tracking of a nonmaneuvering target appears to be superior to previous approaches because of its use of modified polar coordinates (MPC), which reduce the problems with observability, range bias, and covariance ill-conditioning that are encountered with Cartesian coordinates.

Here, the MPC filter is extended to three dimensions by the use of modified spherical coordinates (MSC).[3] In the MSC filter, the six state variables are two angles, their derivatives, inverse range, and range rate over range, which are transformable into Cartesian position and velocity.

This MSC filter differs from that of Ref. 4, which has five state variables, angle plus range measurements, and is apparently restricted to a surface target.

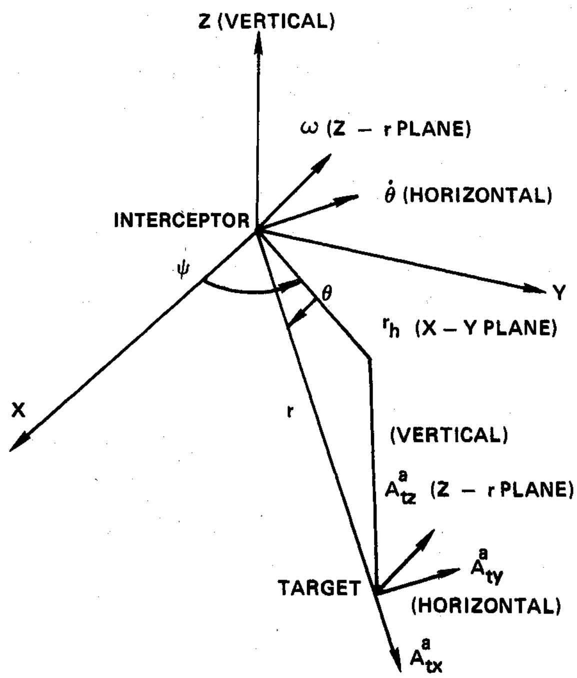

Fig. 1 Basic coordinates for MSC filter.

Presented as Paper 87-2380 at the AIAA Guidance, Navigation, and Control Conference, Monterey, CA, Aug. 17-19, 1987; received Jan. 29, 1989; revision received Nov. 15, 1989.

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